



Entropy generation in turbulent liquid flow through a smooth duct subjected to constant wall temperature

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Abstract

The entropy generation for a fully developed turbulent fluid flow in a smooth duct subjected to constant wall temperature is investigated analytically. The temperature dependence of the viscosity is taken into consideration in the analysis. The ratio of the pumping power to the total heat flux decreases considerably when fluid is heated and the entropy generation per unit heat flux attains a minimum along the duct length for viscous fluids. The results corresponding to the temperature dependent and constant viscosity cases are compared. It is found that constant viscosity assumption may yield a considerable amount of deviation in entropy generation and pumping power results from those of the temperature dependent viscosity cases, especially when highly viscous fluids are considered. © 2000 Elsevier Science Ltd. All rights reserved.

1. Introduction

Heat transfer and fluid pumping power in a heat exchanger are strongly dependent upon the type of fluid flowing through the heat exchanger. It is important to know the fluid properties and their dependence to temperature for a heat exchanger analysis. The temperature changes in the direction of flow and the fluid properties are affected. The temperature variation across the individual flow passages influences the velocity and temperature profiles, and thereby influences the friction factor and the convective heat transfer coefficient [1]. If the thermo-physical properties of the fluids in a heat exchanger vary substantially with temperature, the velocity and the temperature profiles become interrelated and, thus, the heat transfer is affected. Viscosity of a fluid is one of the properties

which is most sensitive to temperature. In the majority of cases, viscosity becomes the only property which may have considerable effect (more than the effect of thermal conductivity) on the heat transfer and temperature variation and therefore temperature dependence of other thermo-physical properties is often negligible. Heat flux and temperature differences in many heat exchangers are considerably large, and the viscosity of working fluids varies significantly as a function of temperature. For example, when the temperature is increased from 20 to 80°C, the viscosity of engine oil decreases 24 times, the viscosity of water decreases 2.7 times and the viscosity of air increases 1.4 times. Therefore, selection of the type of fluid and the range of operating temperatures are very important in the design and performance calculations of a heat exchanger.

On the other hand, the irreversibilities associated with fluid flow through a duct are usually related to heat transfer and viscous friction. The contributions of various mechanisms and design features on the differ-

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Nomenclature

b	dimensional constant in Eq. (15)	T_w	wall temperature of the duct (K)
B	constant in Eq. (19)	\bar{U}	fluid bulk velocity (m/s)
C_p	specific heat capacity (J/kg K)	x	axial distance (m)
D	diameter (m)	ΔT	increase of fluid temperature (K)
Ec	Eckert number [$\bar{U}^2/C_p T_w$]	μ	viscosity (N/s m ²)
f	friction factor	μ_b	viscosity of fluid at bulk temperature (N/s m ²)
\bar{h}	average heat transfer coefficient (W/m ² K)	μ_{ref}	viscosity of fluid at reference temperature (N/s m ²)
$\bar{h}_{c.p.}$	constant property average heat transfer coefficient (W/m ² K)	μ_w	viscosity of fluid at wall temperature (N/s m ²)
k	thermal conductivity (W/m K)	λ	dimensionless axial distance [L/D]
L	length of the duct (m)	Π_1	modified Stanton number [$St\lambda$]
\dot{m}	mass flow rate (kg/s)	Π_2	dimensionless group [$\bar{f}Ec/St$]
n	constant in Eq. (19)	ψ	dimensionless entropy generation [$\dot{S}_{gen}/[\dot{Q}/(T_w-T_o)]$]
\bar{Nu}	average Nusselt number [$\bar{h}D/k$]	ψ'	modified dimensionless entropy generation [$\dot{S}_{gen}/(\dot{Q}/\Delta T)$]
P	pressure (N/m ²)	ρ	density (kg/m ³)
\mathcal{P}_r	pumping power to heat transfer rate ratio	τ	dimensionless inlet wall-to-fluid temperature difference [$(T_w-T_o)/T_w$]
\dot{Q}	total heat transfer rate (W)	θ	dimensionless temperature [$(T-T_w)/(T_o-T_w)$]
Re	Reynolds number [$\rho\bar{U}D/\mu$]		
s	entropy (J/kg K)		
\dot{S}_{gen}	entropy generation (W/K)		
St	Stanton number [$\bar{h}/(\rho\bar{U}C_p)$]		
T	temperature (K)		
T_o	inlet fluid temperature (K)		
T_{ref}	reference temperature (= 293 K)		

ent irreversibility terms often compete with one another [2]. Therefore, there may exist an optimal thermodynamic design which minimizes the amount of entropy generation. For a given thermodynamic system, a set of thermodynamic parameters which optimize operating conditions could be obtained. Dependence of viscosity on the temperature affects not only the viscous friction and pressure drop, but also the heat transfer. This implies that the irreversibilities associated with the heat transfer and viscous friction are also affected.

Bejan [3,4] outlined the second-law analysis in heat transfer and thermal design in detail. He presented the basic steps for the procedure of entropy generation minimization at the system-component level. Nag and Mukherjee [5] studied the thermodynamic optimization of convective heat transfer through a duct with constant wall temperature. In their study, they plotted the variation of entropy generation with the difference of bulk flow inlet and the surface temperatures using a duty parameter. In the case they considered, the duty parameter was also a function of this temperature difference through the heat transfer. They studied the effect of the inlet and the wall temperature difference for small values of this temperature difference.

Heat transfer enhancement techniques that are used to increase the rate of heat transfer are known to

increase the friction factor. Minimization of the total entropy generation for two typical heat transfer enhancement problems related to the variation of heat transfer area and the variation of temperature difference, was studied by Perez-Blanco [6].

In a recent study, Sahin [7] studied the entropy generation in a laminar viscous flow through a duct with constant wall temperature. He showed that there could be an optimum size of heat exchanger and/or inlet temperature of fluid for which the total irreversibility due to heat transfer and pressure drop becomes the minimum. Thus, as an extension to Sahin's work [7], the effect of temperature dependent viscosity during a heating process in a turbulent flow has been investigated in this work for a more accurate determination of entropy generation and required pumping power. The influence of the viscosity variation on the heat transfer coefficient and friction factor was considered. Results for the case of constant viscosity assumption were also included for comparison.

2. Viscosity dependence on temperature

It is evident from experiments that the viscosity of liquids varies considerably with temperature. Around room temperature (293 K), for instance, a 1% change

in temperature produces a 7% change in the viscosity of water and approximately a 26% change in the viscosity of glycerol [8].

For the first approximation, the variation of viscosity with temperature can be assumed to be linear,

$$\mu(T) = \mu_{\text{ref}} - b(T - T_{\text{ref}}) \quad (1)$$

where b is a fluid dependent dimensional constant and T_{ref} is a reference temperature of 293 K. This would be a reasonable approximation if the variation of the bulk temperature is small. However, for highly viscous liquids the variation of viscosity with temperature is exponential and a more accurate empirical correlation of liquid viscosity with the temperature is given by Sherman [8] as

$$\mu(T) = \mu_{\text{ref}} \left(\frac{T}{T_{\text{ref}}} \right)^n \exp \left[B \left(\frac{1}{T} - \frac{1}{T_{\text{ref}}} \right) \right] \quad (2)$$

where n and B are fluid dependent constant parameters. In the present work, both the linear and the exponential viscosity models were used. In addition a constant viscosity model (in which the numerical value of viscosity at an average bulk temperature is taken as constant) is also included in order to see the significance of the variation of viscosity on the results.

3. Temperature variation along the duct

Let us consider the constant cross-sectional area duct shown in Fig. 1. The surface temperature of the duct is kept constant at T_w . An incompressible viscous fluid with mass flow rate, \dot{m} , and inlet temperature, T_o enters the duct of length L . The density ρ , thermal conductivity k , and specific heat C_p of the fluid are assumed to be constant within the range of temperatures considered in this study (Table 1). Heat transfer to the bulk of the fluid occurs through the average heat transfer coefficient, \bar{h} , which is not constant but is a function of the viscosity variation. The effect of viscosity on the average heat transfer coefficient is given by [9]:

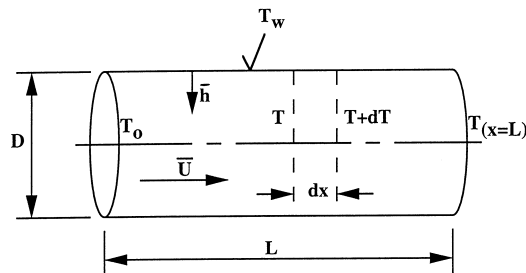


Fig. 1. Schematic view of duct.

$$\frac{\bar{h}}{\bar{h}_{c.p.}} = \frac{\bar{Nu}}{Nu_{c.p.}} = \left(\frac{\mu_b}{\mu_w} \right)^n \quad (3)$$

where the exponent n is equal to 0.11 for heating and 0.25 for cooling. For a fully developed turbulent flow, $\bar{h}_{c.p.}$ is given by Gnielinski Eq. [10]:

$$\bar{h}_{c.p.} = \frac{k}{D} \bar{Nu}_{c.p.} = \frac{k}{D} \frac{(\bar{f}/8)(Re - 1000)Pr}{1 + 12.7(Pr^{2/3} - 1)\sqrt{\bar{f}/8}} \quad (4)$$

On the other hand, the average Darcy friction factor, \bar{f} , for this smooth duct is also considered to be a function of temperature dependent viscosity and is given by [9]:

$$\frac{\bar{f}}{\bar{f}_{c.p.}} = \left(\frac{\mu_b}{\mu_w} \right)^{-0.25} \quad (5)$$

where the friction factor for constant properties is given by [10]:

$$\bar{f}_{c.p.} = [0.79 \ln(Re) - 1.64]^{-2} \quad (6)$$

To account for the variation of the bulk temperature along the duct length, μ_b and therefore Re and Pr , in Eqs. (3) and (6), are related to the bulk fluid temperature halfway between the inlet and outlet of the duct, as suggested by Kreith and Bohn [11]. Since the temperature variation along the duct is initially unknown and depends on \bar{h} , a trial and error procedure is followed to determine both \bar{h} and \bar{f} .

The rate of heat transfer to the fluid inside the control volume shown in Fig. 1 is

$$\delta \dot{Q} = \dot{m} C_p dT = \bar{h} \pi D (T_w - T) dx \quad (7)$$

where

Table 1
Thermophysical properties and parameters used

	Water	Glycerol
b (N s/m ² K)	8.9438×10^{-6}	0.0182
B	4700	23100
C_p (J/kg K)	4182	2428
D (m)	0.1	1.0
k (W/m K)	0.6	0.264
n	8.9	52.4
T_{ref} (K)	293	293
T_w (K)	373	373
\bar{U} (m/s)	0.02	10
μ_{ref} (N s/m ²)	9.93×10^{-4}	1.48
Π_1	0.0–0.8	0.0–0.8
ρ (kg/m ³)	998.2	1260
τ	0.0–0.25	0.0–0.25

$$\dot{m} = \rho \bar{U} \frac{\pi D^2}{4}.$$

It should be noted that in writing Eq. (7), the duct is assumed to have a circular cross section, however the analysis is not affected assuming cross sectional areas other than circular.

Integrating Eq. (7), the bulk temperature variation of the fluid along the duct can be obtained as:

$$T = T_w - (T_w - T_o) \exp \left[- \frac{4\bar{h}}{\rho \bar{U} D C_p} x \right]. \quad (8)$$

The temperature variation along the duct approaches the duct wall temperature exponentially assuming a uniform heat transfer coefficient evaluated at the bulk temperature halfway between the inlet and outlet of the duct.

Eq. (8) can be re-written as:

$$\theta = \exp \left(- 4 \frac{St}{D} x \right) \quad (9)$$

where θ is the dimensionless temperature defined as,

$$\theta = \frac{T - T_w}{T_o - T_w}$$

and St is the Stanton number defined as

$$St = \frac{\bar{h}}{\rho \bar{U} C_p}.$$

4. The total entropy generation

The total entropy generation within the control volume in Fig. 1 can be written as:

$$d\dot{S}_{gen} = \dot{m} ds - \frac{\delta \dot{Q}}{T_w} \quad (10)$$

where, for an incompressible fluid,

$$ds = \frac{C_p dT}{T} - \frac{dP}{\rho T}.$$

Substituting Eq. (7) in Eq. (10), the total entropy generation becomes

$$d\dot{S}_{gen} = \dot{m} C_p \left[\frac{T_w - T}{TT_w} dT - \frac{1}{\rho C_p T} dP \right]. \quad (11)$$

The pressure drop in Eq. (11) is given by Kreith and Bohn [11]

$$dP = - \frac{\bar{f} \rho \bar{U}^2}{2D} dx. \quad (12)$$

Integrating Eq. (11) along the duct length, L , using Eqs. (8) and (12), the total entropy generation is obtained as

$$\dot{S}_{gen} = \dot{m} C_p \left\{ \ln \left[\frac{1 - \tau e^{-4St\lambda}}{1 - \tau} \right] - \tau (1 - e^{-4St\lambda}) + \frac{1}{8} \bar{f} \frac{Ec}{St} \ln \left[\frac{e^{4St\lambda} - \tau}{1 - \tau} \right] \right\} \quad (13)$$

where τ is the dimensionless temperature difference

$$\tau = \frac{T_w - T_o}{T_w},$$

λ is the dimensionless length of duct

$$\lambda = \frac{L}{D},$$

and Ec is the Eckert number defined as

$$Ec = \frac{\bar{U}^2}{C_p T_w}.$$

The total rate of heat transfer to the fluid is obtained by integrating Eq. (7) along the duct length and can be written as:

$$\dot{Q} = \dot{m} C_p (T_w - T_o) (1 - e^{-4St\lambda}). \quad (14)$$

Now defining a dimensionless entropy generation as:

$$\psi = \frac{\dot{S}_{gen}}{\dot{Q}/(T_w - T_o)}$$

Eq. (13) can be written as

$$\psi = \frac{1}{1 - e^{-4St\lambda}} \left\{ \ln \left[\frac{1 - \tau e^{-4St\lambda}}{1 - \tau} \right] - \tau (1 - e^{-4St\lambda}) + \frac{1}{8} \bar{f} \frac{Ec}{St} \ln \left[\frac{e^{4St\lambda} - \tau}{1 - \tau} \right] \right\}. \quad (15)$$

Therefore, two dimensionless groups naturally arise in Eq. (15) in fully developed turbulent flow as:

$$\Pi_1 = St\lambda \quad (16)$$

and

$$\Pi_2 = \bar{f} \frac{Ec}{St}. \quad (17)$$

Thus Eq. (15) can be written in a compact form for the constant viscosity assumption as:

$$\psi = \frac{1}{1 - e^{-4\Pi_1}} \left\{ \ln \left[\frac{1 - \tau e^{-4\Pi_1}}{1 - \tau} \right] - \tau(1 - e^{-4\Pi_1}) + \frac{1}{8} \Pi_2 \ln \left[\frac{e^{4\Pi_1} - \tau}{1 - \tau} \right] \right\} \quad (18)$$

which is a function of three nondimensional parameters, namely τ , Π_1 and Π_2 . Among these parameters, τ represents the fluid inlet temperature T_o and Π_1 represents the duct length L . Once the type of the fluid and the mass flow rate are fixed, the parameter Π_2 can be calculated based on temperature analysis. Thus, τ and Π_1 are the two design parameters that can be varied for determining the effects of duct length and/or inlet fluid temperature on the entropy generation.

It should be noted that the dimensionless entropy generation, ψ , in the above analysis is a function of the total heat transfer rate, \dot{Q} , which in turn depends on the length of the duct and inlet fluid temperature. However, a modified dimensionless entropy generation can be defined on the basis of unit heat capacity rate of fluid in the duct as:

$$\psi' = \frac{\dot{S}_{gen}}{\dot{m}C_p} = \frac{\dot{S}_{gen}}{(\dot{Q}/\Delta T)} \quad (19)$$

where ΔT is the increase of the bulk temperature of the fluid in the duct, $\Delta T = T(L) - T_o$. Noting that $\dot{Q}/\Delta T = \dot{m}C_p$ is constant for fixed mass flow rate and

$$\psi' = (1 - e^{-4\Pi_1})\psi. \quad (20)$$

ψ' indicating the total entropy generation along the duct, is expected to increase with the increase in duct length.

5. Pumping power to heat transfer rate ratio

The pumping power to heat transfer rate ratio is

$$\mathcal{P}_r = \frac{(\pi D^2/4)\bar{U}\Delta P}{\dot{Q}} \quad (21)$$

Using Eqs. (12) and (14), the pumping power to heat transfer rate ratio, \mathcal{P}_r for fully developed turbulent

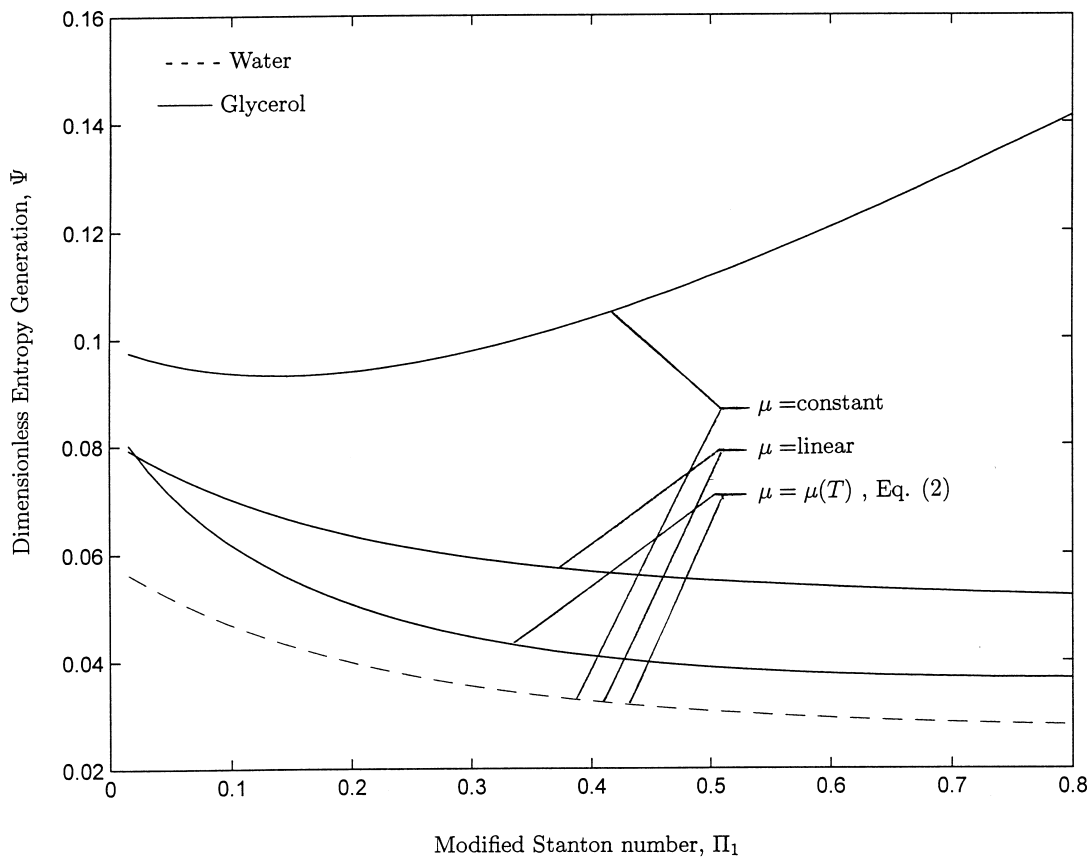


Fig. 2. Dimensionless entropy generation, ψ , vs modified in Stanton number, Π_1 , for water and glycerol with three cases of viscosity dependence (effect of viscosity variation with temperature for water is negligible). $\tau = 0.214$.

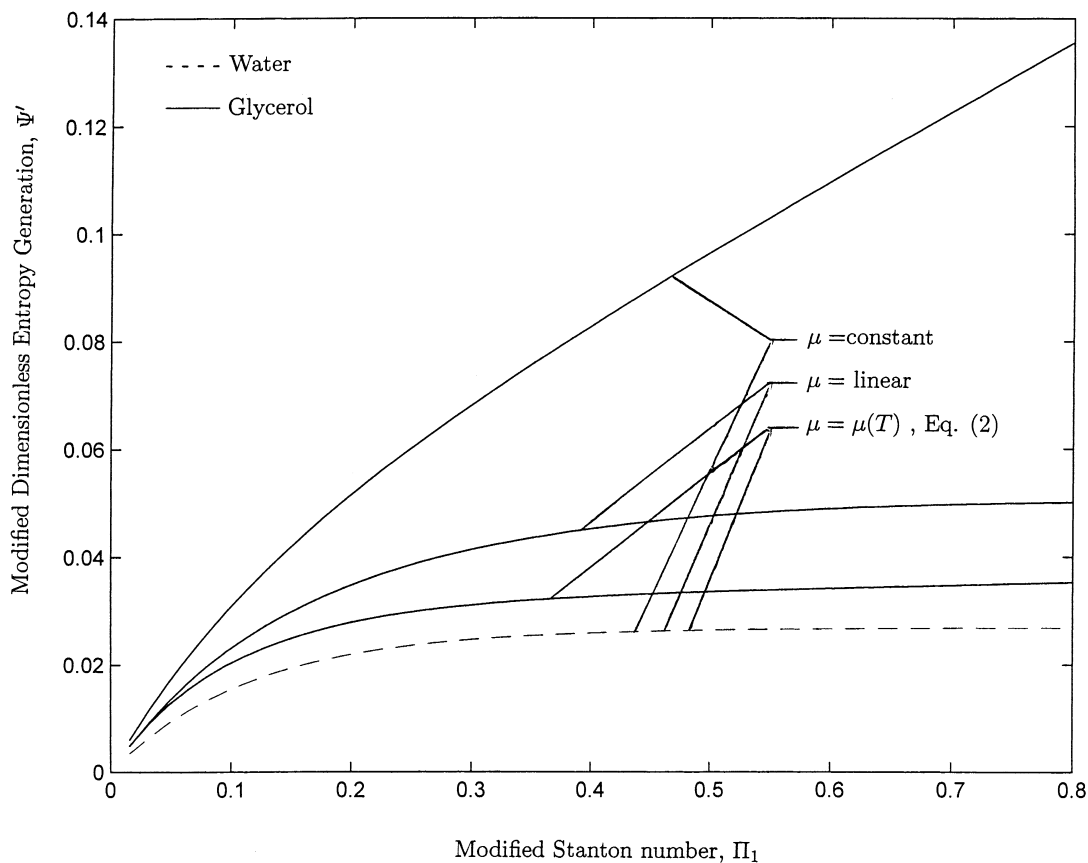


Fig. 3. Modified dimensionless entropy generation, ψ' , vs modified Stanton number, Π_1 , for water and glycerol with three cases of viscosity dependence (effect of viscosity variation with temperature for water is negligible). $\tau=0.214$.

flow is obtained as

$$\mathcal{P}_r = \frac{1}{2} \frac{\Pi_1 \Pi_2}{\tau(1 - e^{-4\Pi_1})}. \quad (22)$$

6. Discussion

The primary concern in a heat exchanger design is the amount of heat transfer. However, the second law efficiency can be very low during the heat exchange process, because of the large amount of entropy generation. This needs to be minimized for efficient utilization of energy. Thus, entropy generation per unit amount of heat transfer is considered to be a suitable quantity in dealing with the second law analysis of a duct. In the present analysis this ratio is represented by the dimensionless entropy generation, ψ , defined as the entropy generation per unit heat transfer rate for a specified duct inlet and wall temperatures.

In order to show the effect of viscosity on the total

entropy generation, two incompressible fluids, namely water and glycerol were selected. The parameters and the thermophysical properties used in the numerical example are given in Table 1. The wall temperature, the velocity of the incompressible fluid, and the cross sectional area of the duct were fixed. The length of the duct and the inlet fluid temperature are left as variables to be studied. During the computations, Re numbers were calculated to make sure that fully developed turbulent flow is maintained within the range of parameters used. The convergence criterion used during the evaluation of viscosity dependent heat transfer coefficient \bar{h} and friction factor \bar{f} through Eqs. (3) and (8) is $|(T_k - T_{k-1})| < \epsilon$, where $\epsilon=0.1$ K and k is the iteration counter.

Fig. 2 shows the variation of dimensionless entropy generation, ψ , vs modified Stanton number, Π_1 , for water and glycerol with three cases of viscosity dependence. Since the viscosity of water is low, the three curves corresponding to constant, linear, and variable viscosity dependence on temperature are essentially the same. This means that the last term in Eq. (18) is neg-

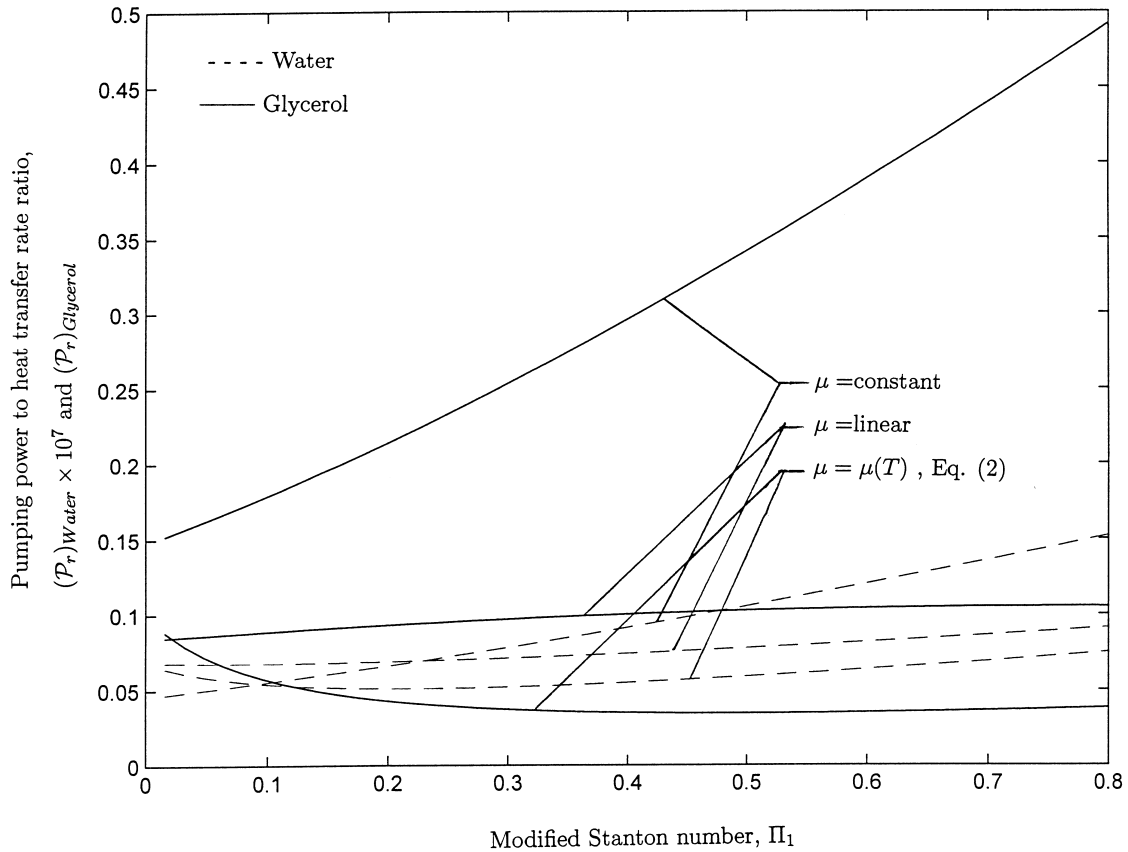


Fig. 4. Pumping power to heat transfer rate ratio, \mathcal{P}_r vs modified Stanton number, Π_1 , for water and glycerol with three cases of viscosity dependence. $\tau = 0.214$.

ligible for water. For glycerol, however, the effect of the assumed variation of viscosity on ψ , is apparent as shown in Fig. 2. The dimensionless entropy generation, ψ , calculated based on the constant viscosity assumption, yields considerably higher values than those calculated for temperature dependent viscosity. Since Π_1 represents the length of the duct, the dimensionless entropy generation defined on the basis of total heat transfer rate to the duct, ψ , decreases initially and then starts increasing along the duct length. The rate of increase in entropy generation approaches a constant value as the total heat transfer rate to the fluid approaches its maximum value of

$$\dot{Q}_{\max} = \dot{m}C_p(T_w - T_o).$$

For long ducts where $e^{-4\Pi_1} \ll 1$ and $e^{4\Pi_1} \gg \tau$, it can be shown from Eq. (18) that the entropy generation increases linearly with the slope

$$\frac{d\psi}{d\Pi_1} = \frac{\Pi_2}{2}$$

for the case of constant viscosity.

Fig. 3 shows the modified dimensionless entropy generation defined based on the unit heat capacity rate along the duct, ψ' , as a function of modified Stanton number, Π_1 , for water and glycerol for the three cases of viscosity dependence. The effect of viscosity is negligible in the case of water. However apparent in the case of glycerol as shown in Fig. 3, ψ' adds up along the duct length and shows an increase in general. As in the case of ψ , the assumption of constant viscosity yields considerably higher values of ψ' compared with those for temperature dependent viscosity. ψ' and ψ differ only for small values of Π_1 . For large values of Π_1 , $\psi' = \psi$ as can be seen from Eq. (20).

Variation of the pumping power to heat transfer rate ratio, \mathcal{P}_r , with Π_1 is shown in Fig. 4 for water and glycerol. In both cases, the constant viscosity assumption yields unreasonably high pumping power ratios as expected. Due to an increase in the bulk tem-

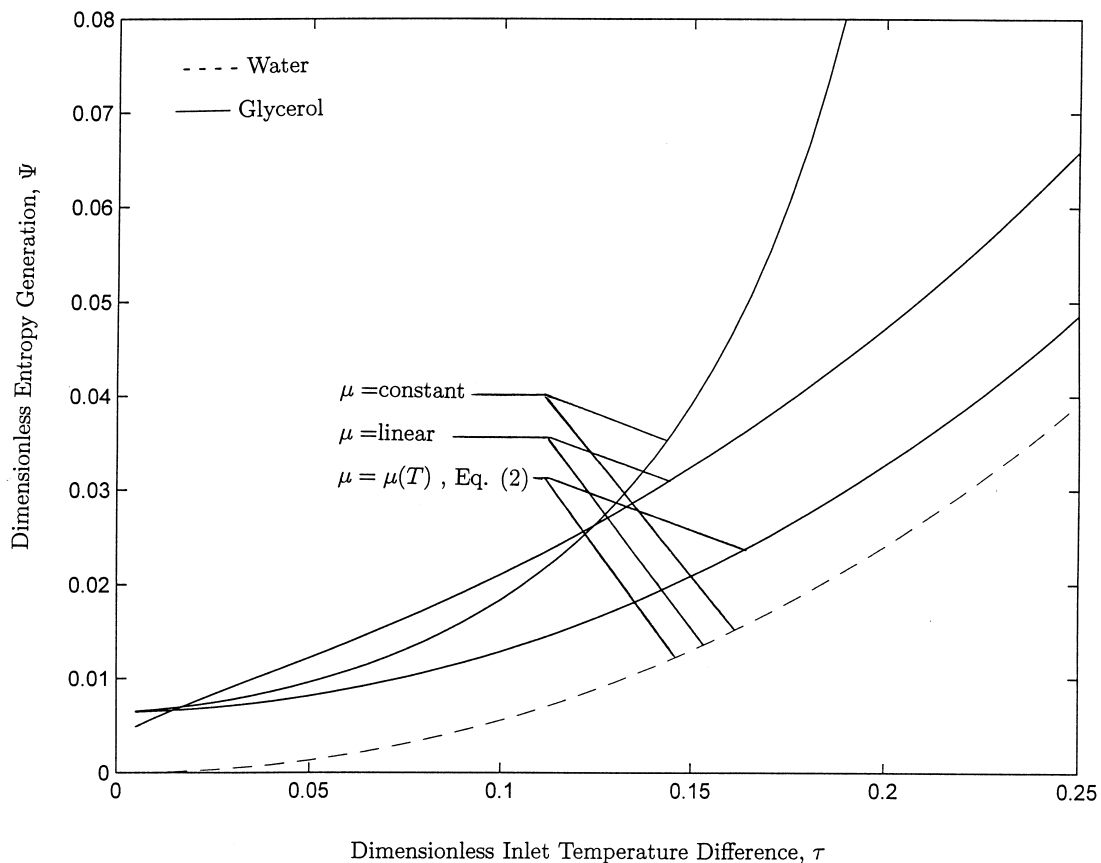


Fig. 5. Dimensionless entropy generation, ψ , vs dimensionless inlet wall-to-fluid temperature difference, τ , for water and glycerol with three cases of viscosity dependence (effect of viscosity variation with temperature for water is negligible). $\Pi_1 = 0.8$.

perature and a decrease in viscosity, the pumping power to heat transfer rate ratio may decrease initially and then increase, as the total heat transfer rate to the fluid decreases as the bulk temperature approaches the wall temperature as Π_1 is increased. For large values of Π_1 and constant viscosity, it can be shown from Eq. (22) that the pumping ratio, \mathcal{P}_r , increases linearly with the slope

$$\frac{d\mathcal{P}_r}{d\Pi_1} = \frac{\Pi_2}{2\tau}.$$

For small values of Π_1 the effect of the viscosity variation is small and in the limit,

$$\lim_{\Pi_1 \rightarrow 0} \mathcal{P}_r = \frac{\Pi_2}{8\tau}$$

which is clearly a function of the fluid viscosity.

The variation of the dimensionless entropy generation, ψ with dimensionless inlet wall-to-fluid temperature difference, τ , is shown in Fig. 5 for water and glycerol with the three cases of viscosity dependence.

Since the viscosity of water is low, the three curves corresponding to constant, linear, and variable viscosity dependence on temperature, are essentially the same as those mentioned above. The effect of the assumed variation of viscosity on ψ is apparent in the case of glycerol. The dimensionless entropy generation, ψ , calculated based on the constant viscosity assumption yields considerably higher values than those calculated for temperature dependent viscosity — especially for large values of τ . Since τ represents the difference between the temperature of the duct surface and that of the inlet fluid, the dimensionless entropy generation defined on the basis of total heat transfer rate to the duct, ψ , increases as τ increases due to the increase in the gap between the bulk and wall temperatures. For small values of τ , the total entropy generation is due to the viscous friction; and in the limit when $\tau=0$, the total dimensionless entropy change using Eq. (18) becomes:

$$\psi = \frac{1}{2} \frac{\Pi_1 \Pi_2}{1 - e^{-4\Pi_1}}.$$

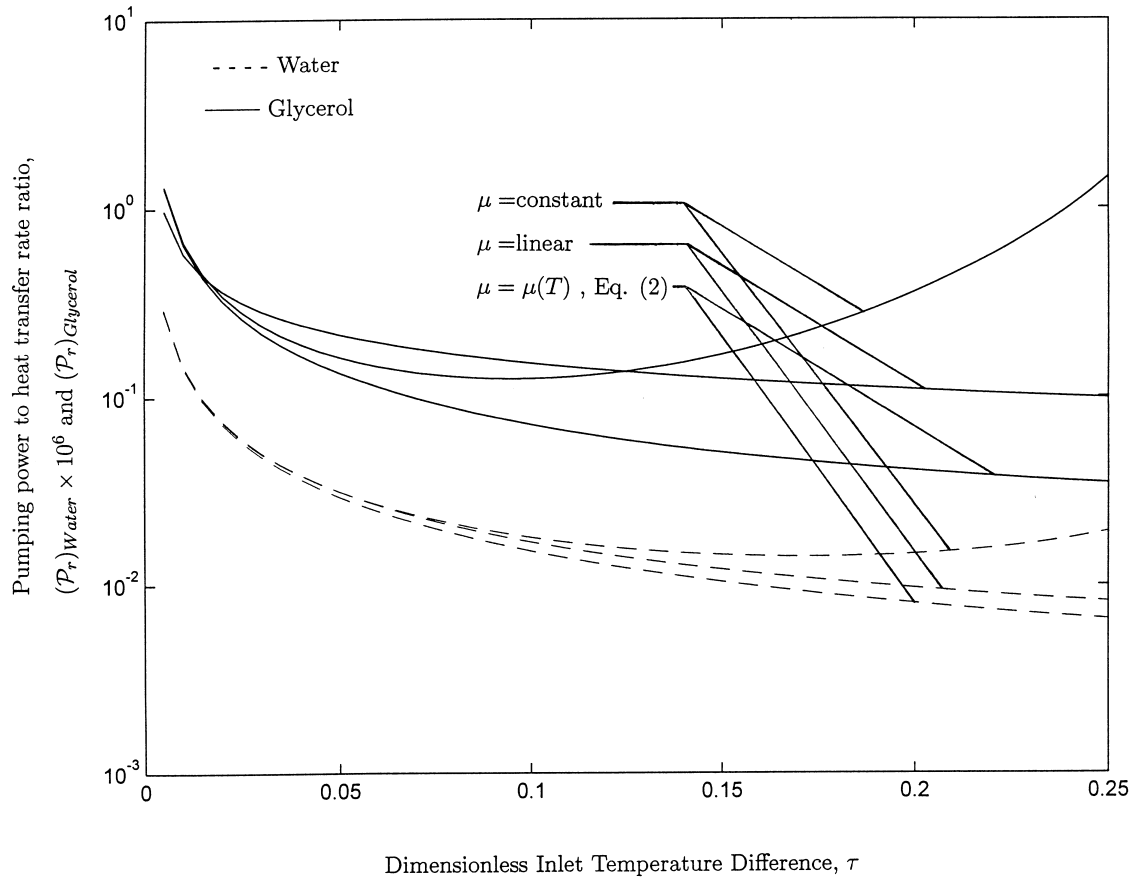


Fig. 6. Pumping power to heat transfer rate ratio, \mathcal{P}_r , vs dimensionless inlet wall-to-fluid temperature difference, τ , for water and glycerol with three cases of viscosity dependence. $\Pi_1 = 0.8$.

The modified dimensionless entropy generation defined, based on the unit heat capacity rate through the duct, ψ' , shows similar behaviour to that of ψ . This was expected since ψ' and ψ are related through a constant factor of $(1 - e^{-4\Pi_1})$ as given in Eq. (20).

Fig. 6 shows the variation of the pumping power to heat transfer rate ratio, \mathcal{P}_r , with respect to the dimensionless inlet wall-to-fluid temperature difference, τ , for water and glycerol. As noted above, the constant viscosity assumption yields higher pumping power ratios in both cases, especially for higher values of τ in which case the viscosity variation is considerable.

7. Conclusions

An analytical study has been performed to investigate the entropy generation for a turbulent viscous flow in a duct subjected to constant wall temperature. The entropy generation was found to be a function of three dimensionless numbers, namely Π_1 , Π_2 and τ .

Entropy generation per unit amount of heat transfer, ψ , decreases initially and then increases along the duct length. However, the pumping power ratio, \mathcal{P}_r , for viscous fluids decreases due to the decrease in viscosity. It was found that entropy generation, ψ , increased with the increase in the dimensionless temperature difference between the inlet fluid and surface temperature, τ . However, the pumping power ratio decreased with an increasing τ .

Constant viscosity assumption yields unreasonably higher values of entropy generation for viscous fluids such as glycerol. In the case of water, the viscosity variation is small, therefore the constant viscosity model gives reasonably accurate values of entropy generation. However, the pumping power results are more sensitive to viscosity dependence on the temperature.

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